



MASSACHUSETTS
GENERAL HOSPITAL

RADIATION ONCOLOGY

Deformable Image Registration Part 2

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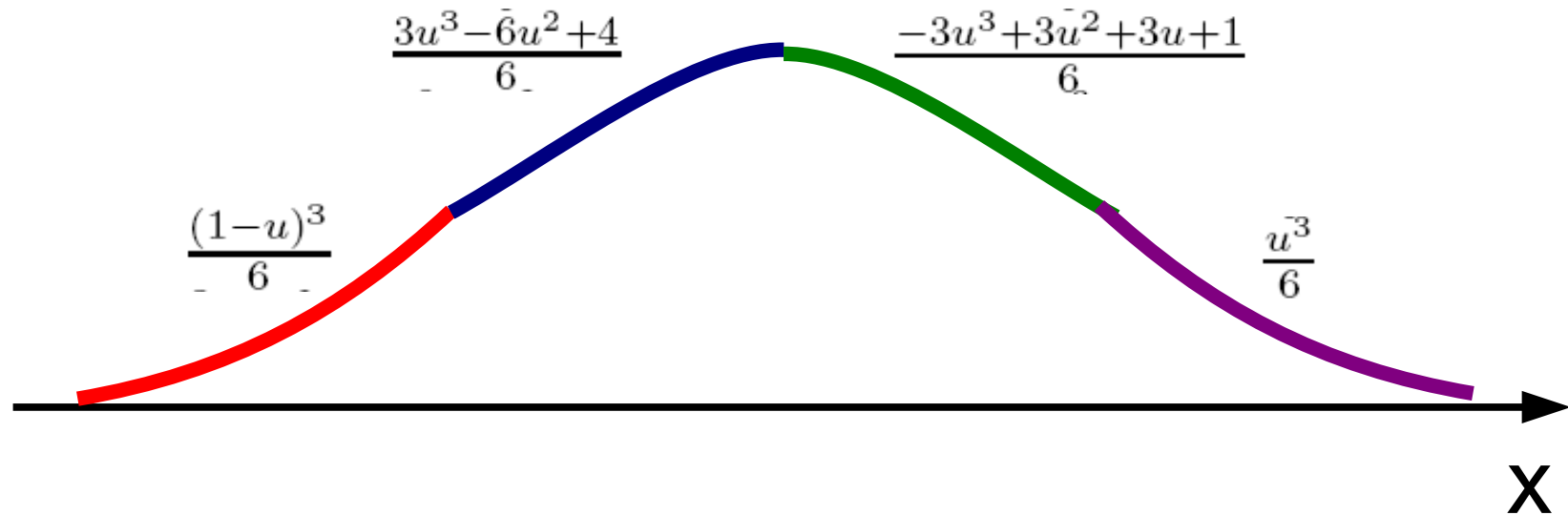
B-splines

- B-spline is short for “basis spline”
- A function is represented as a linear combination of basis functions

$$u(x) = \sum_i p_i \beta_i(x)$$

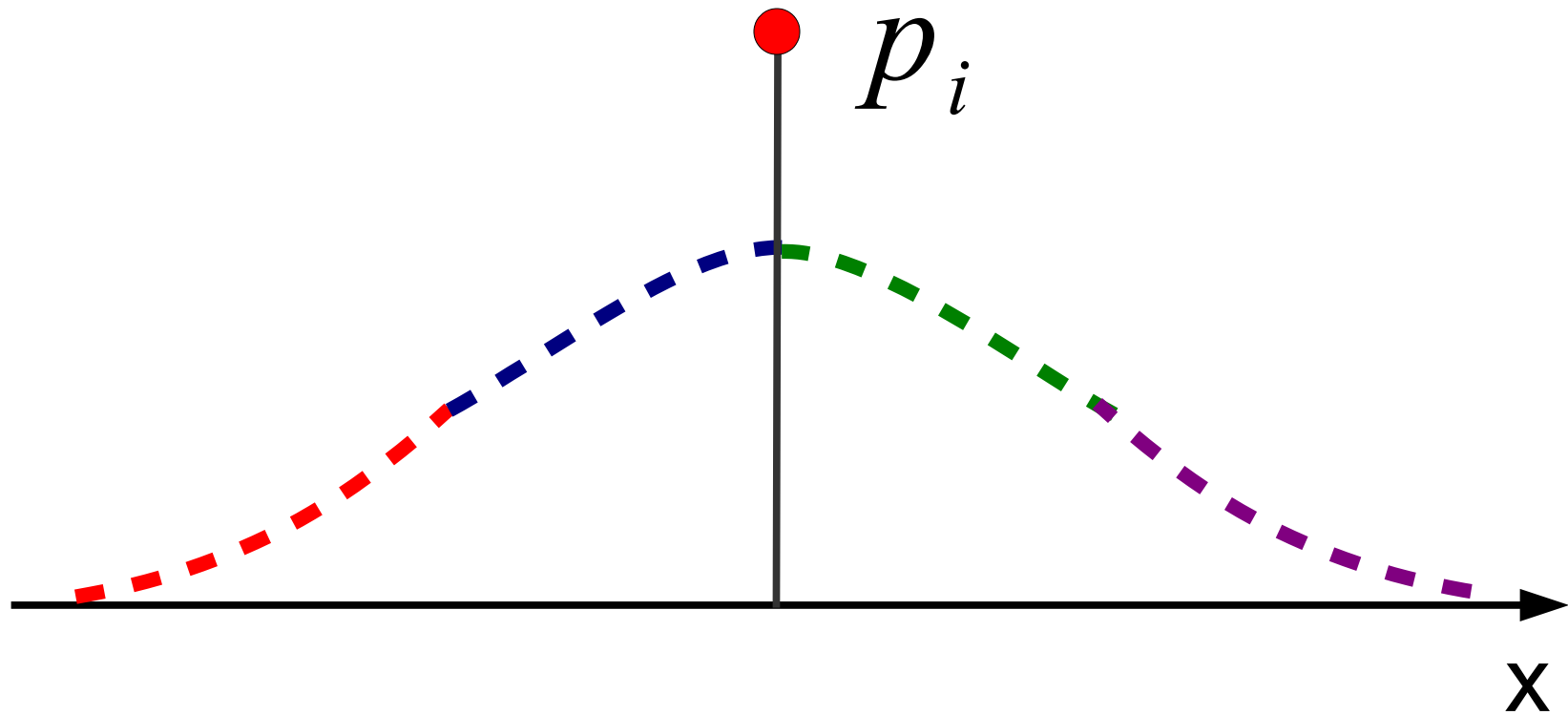
Cubic B-splines

$\beta(x)$ is a piecewise cubic polynomial

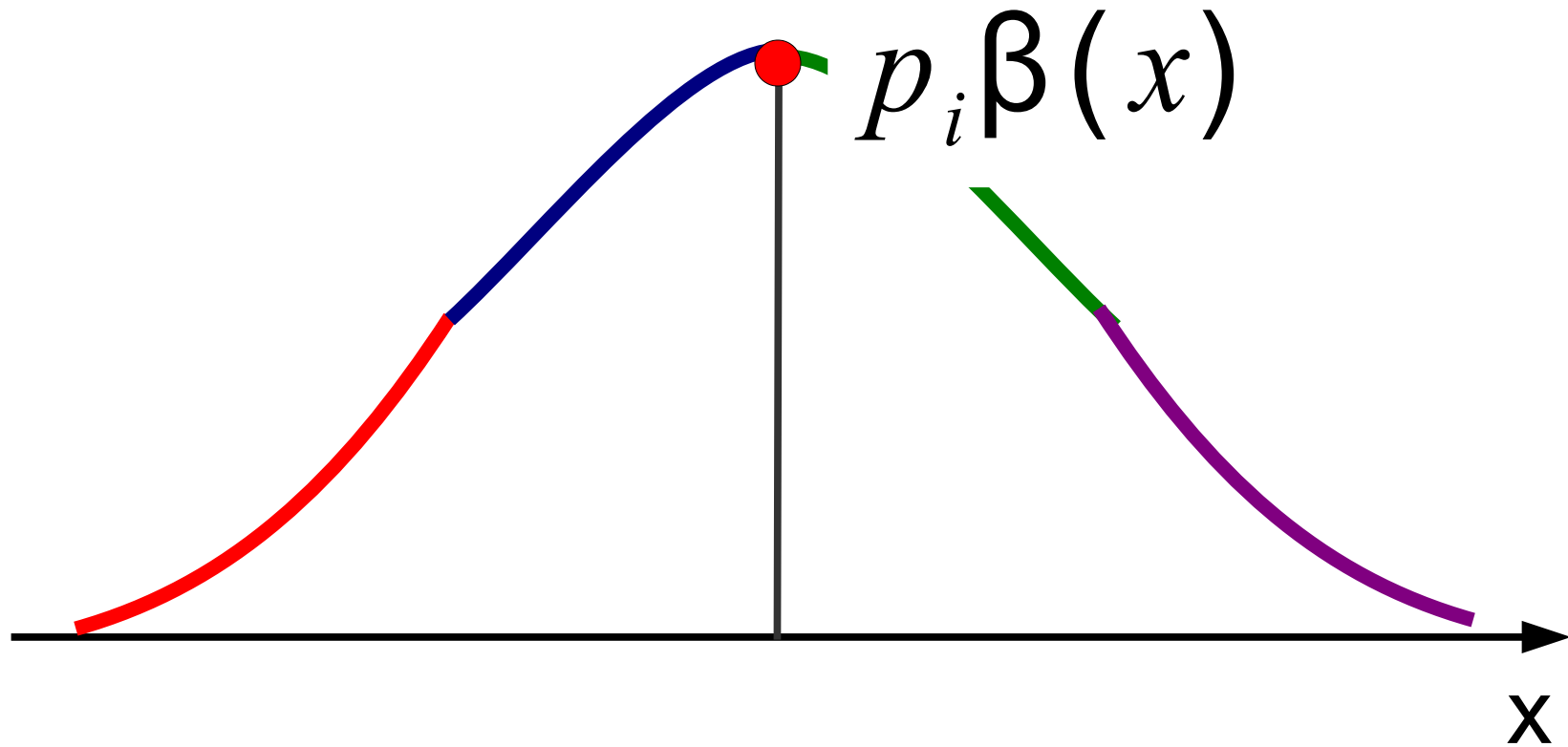


Cubic B-splines

p_i is a scaling factor

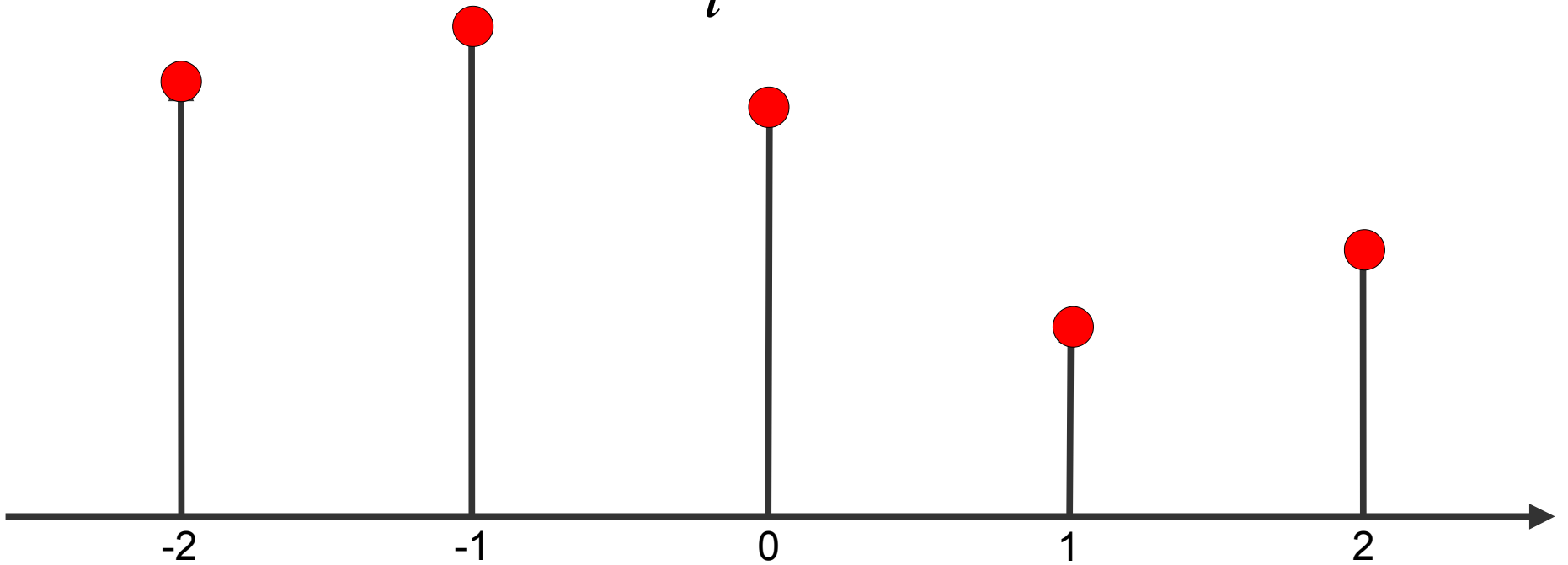


Cubic B-splines



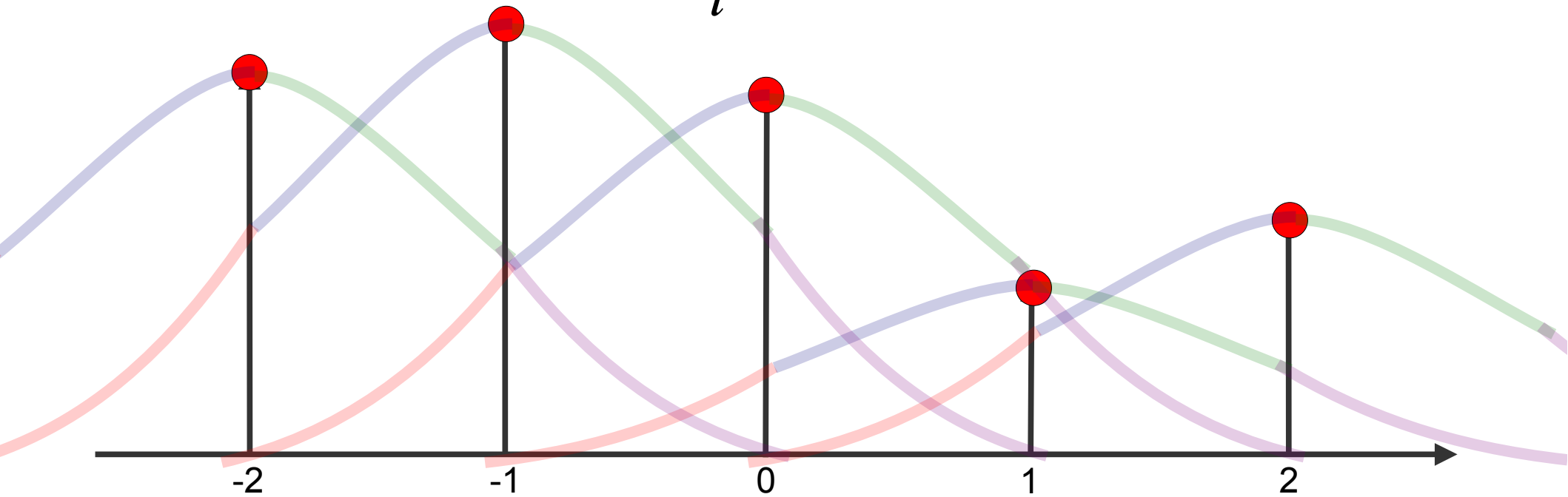
Uniform cubic B-splines

$$v(x) = \sum_i p_i \beta_i(x)$$



Uniform cubic B-splines

$$v(x) = \sum_i p_i \beta_i(x)$$



B-Splines in 3-D

- Three dimensional positions are outer product

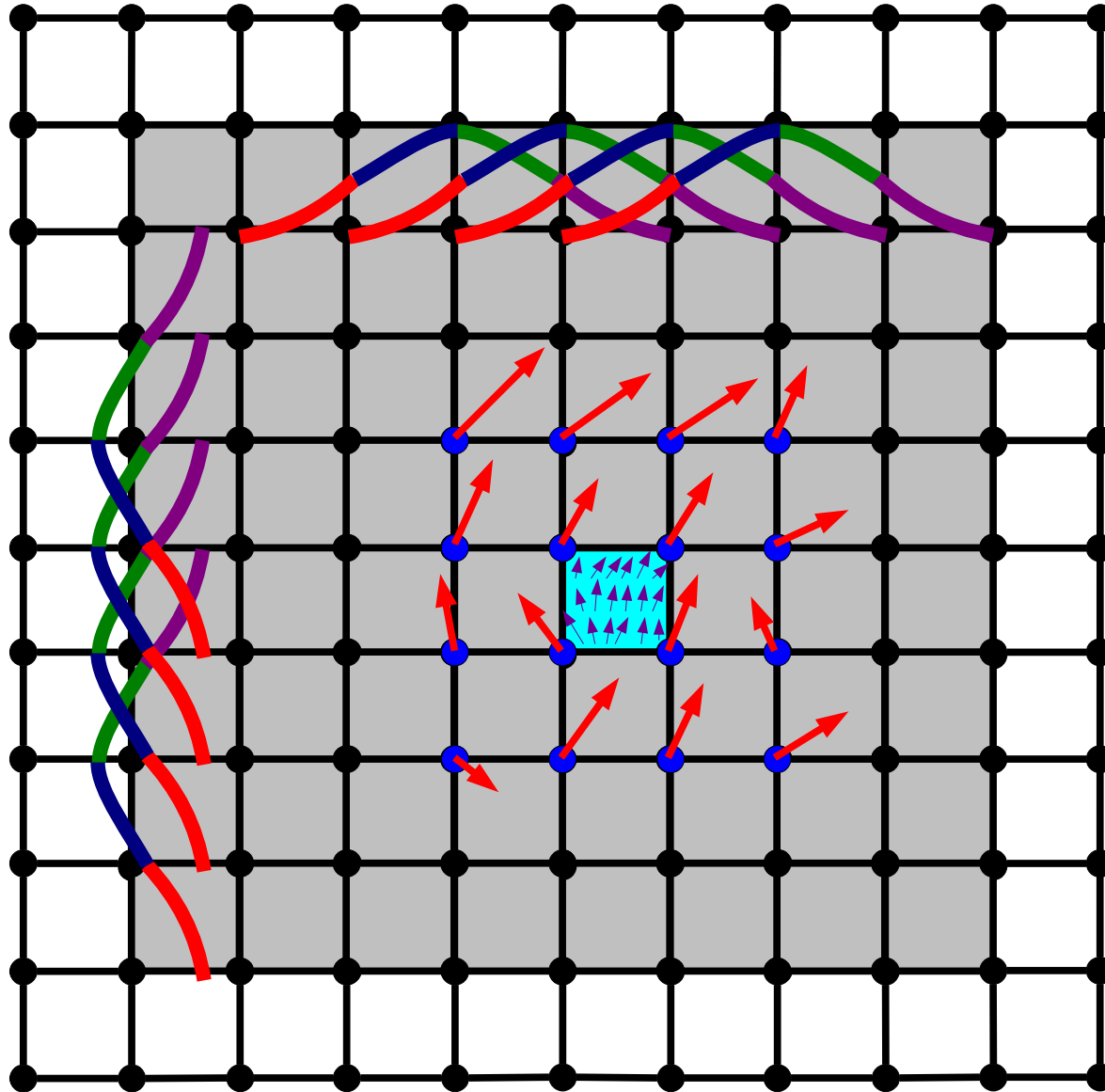
$$t(x, y, z) = \sum_i \sum_j \sum_k p_{i,j,k} \beta_i(x) \beta_j(y) \beta_k(z)$$

B-Splines in 3-D

- Each displacement field is a separate function

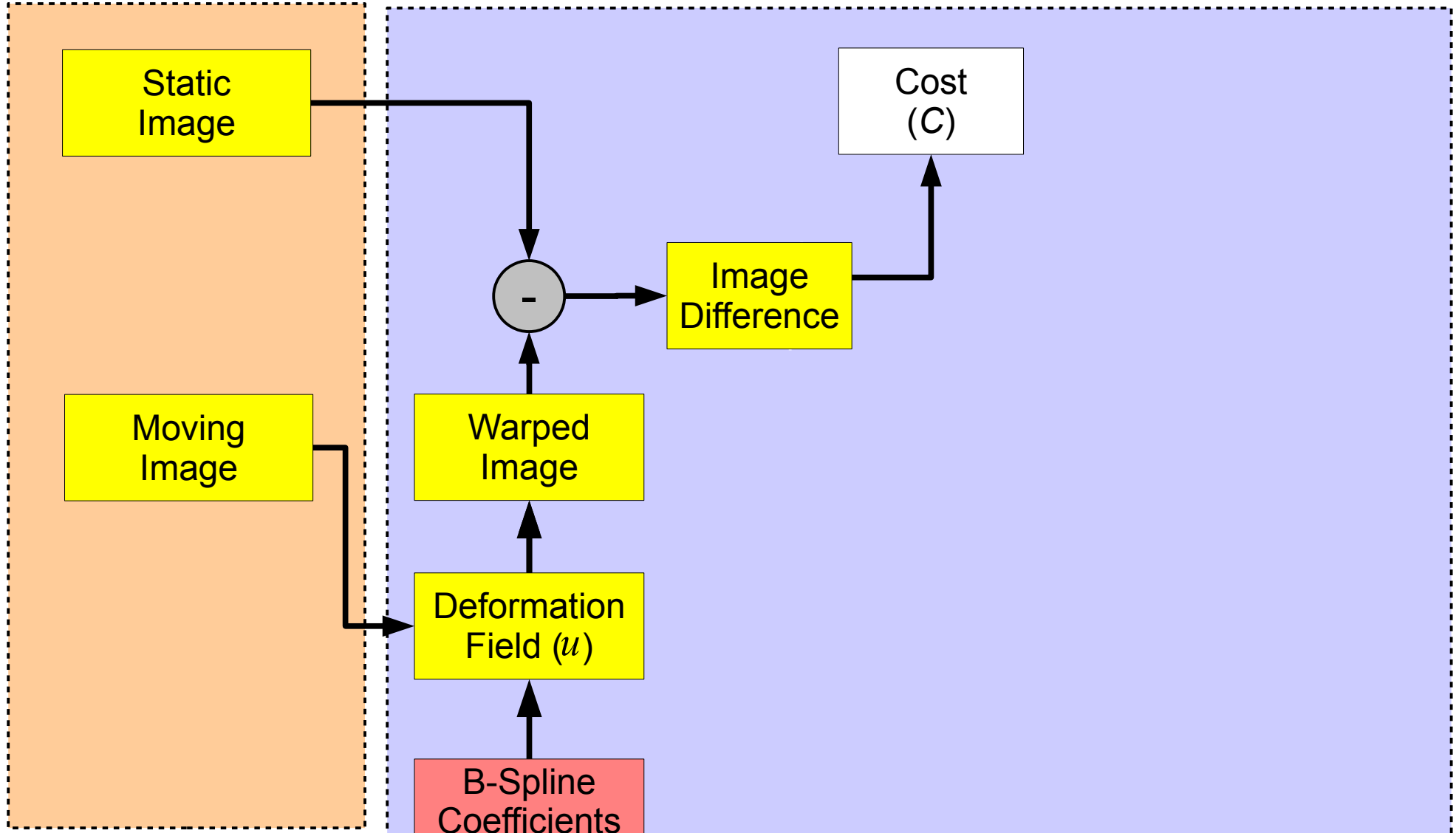
$$t(x) = [t_x, t_y, t_z]$$
$$= \begin{bmatrix} \sum_i \sum_j \sum_k p_{i,j,k}^{(x)} \beta_i(x) \beta_j(y) \beta_k(z) \\ \sum_i \sum_j \sum_k p_{i,j,k}^{(y)} \beta_i(x) \beta_j(y) \beta_k(z) \\ \sum_i \sum_j \sum_k p_{i,j,k}^{(z)} \beta_i(x) \beta_j(y) \beta_k(z) \end{bmatrix}$$





B-splines for vector fields



INPUTS

ITERATIVE REGISTRATION PROCESS



-  Control Point Quantity
-  Voxelized Quantity
-  Optimizer
-  Function

Cost function gradient

- Sum of Squared Difference (SSD)

$$C = \sum_x [F(x) - M(x + u(x))]^2$$

- We want to optimize the B-spline coefficients:

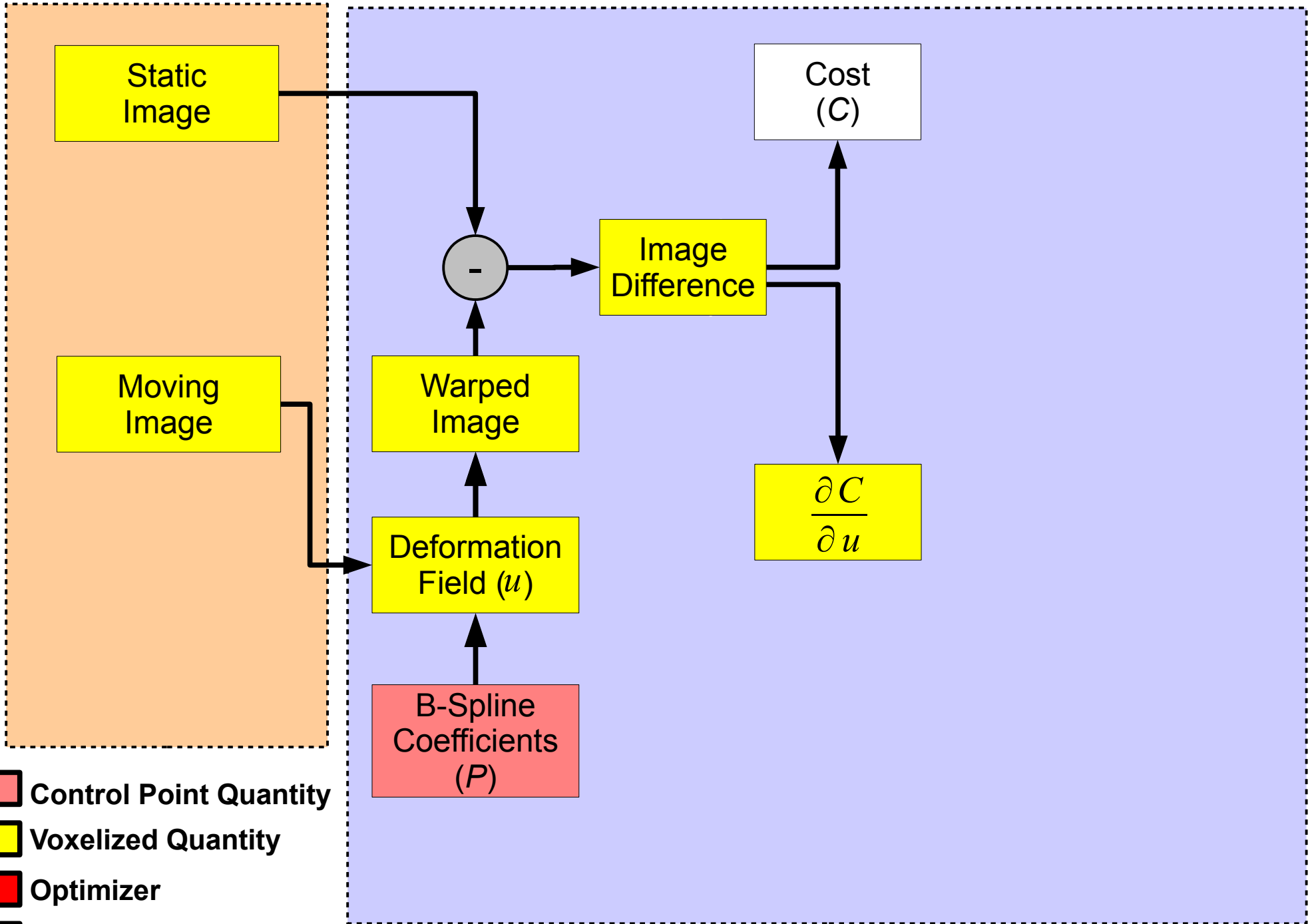
$$u(x) = \sum_i p_i \beta_i(x)$$

- Need the gradient of C with respect to P :

$$\frac{\partial C}{\partial P} = \frac{\partial C}{\partial u} \frac{\partial u}{\partial P}$$

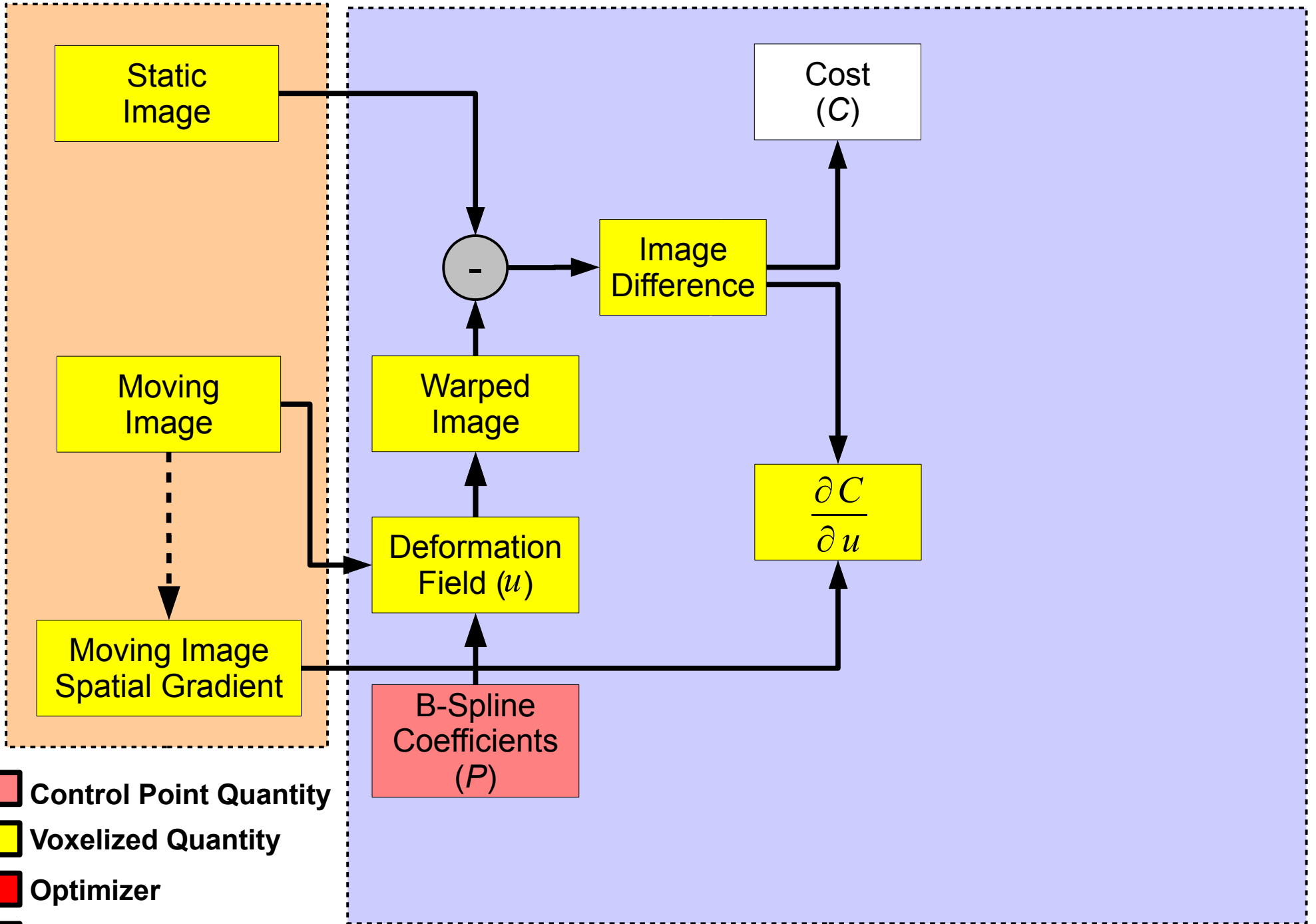
INPUTS

ITERATIVE REGISTRATION PROCESS



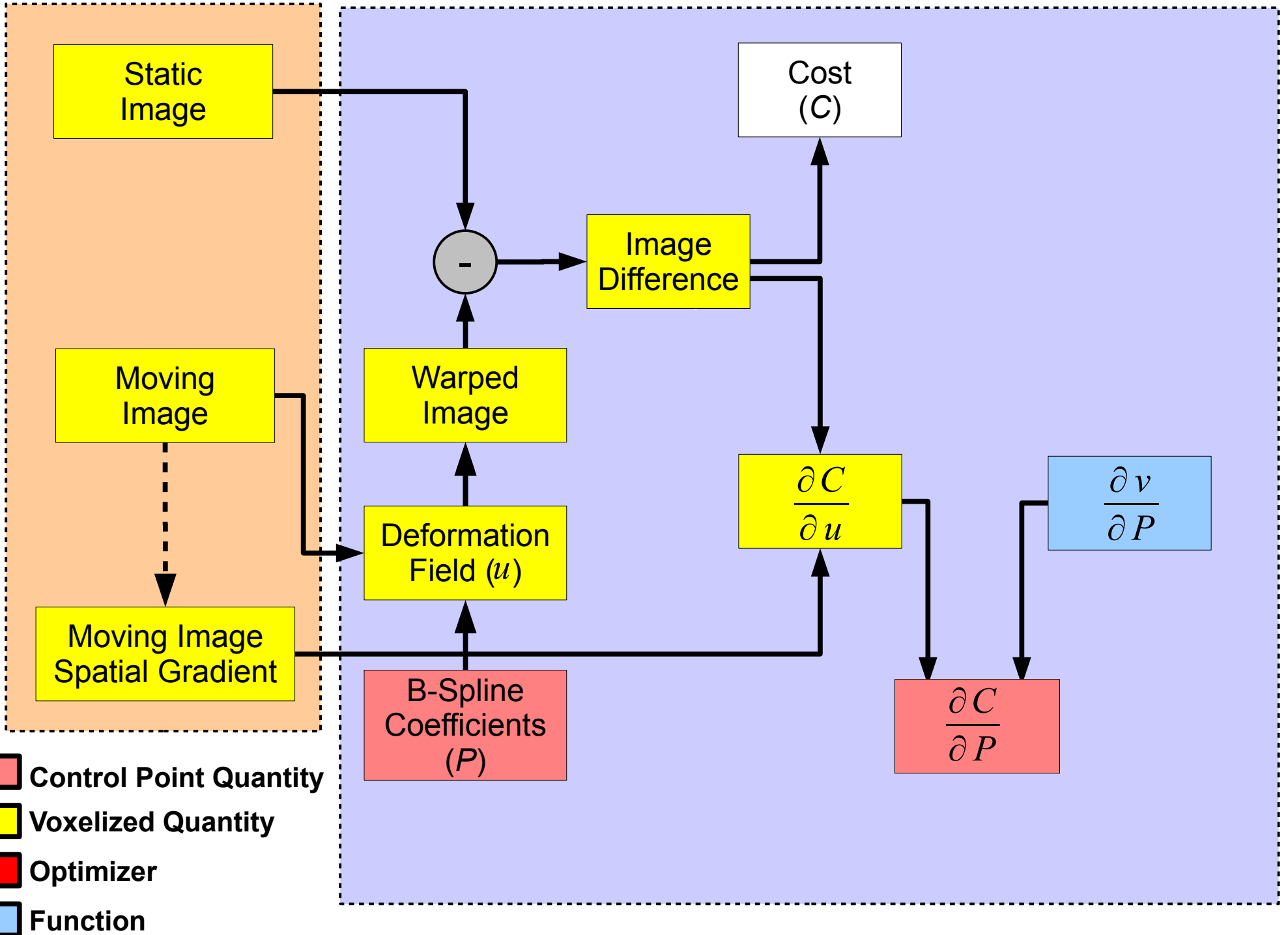
INPUTS

ITERATIVE REGISTRATION PROCESS



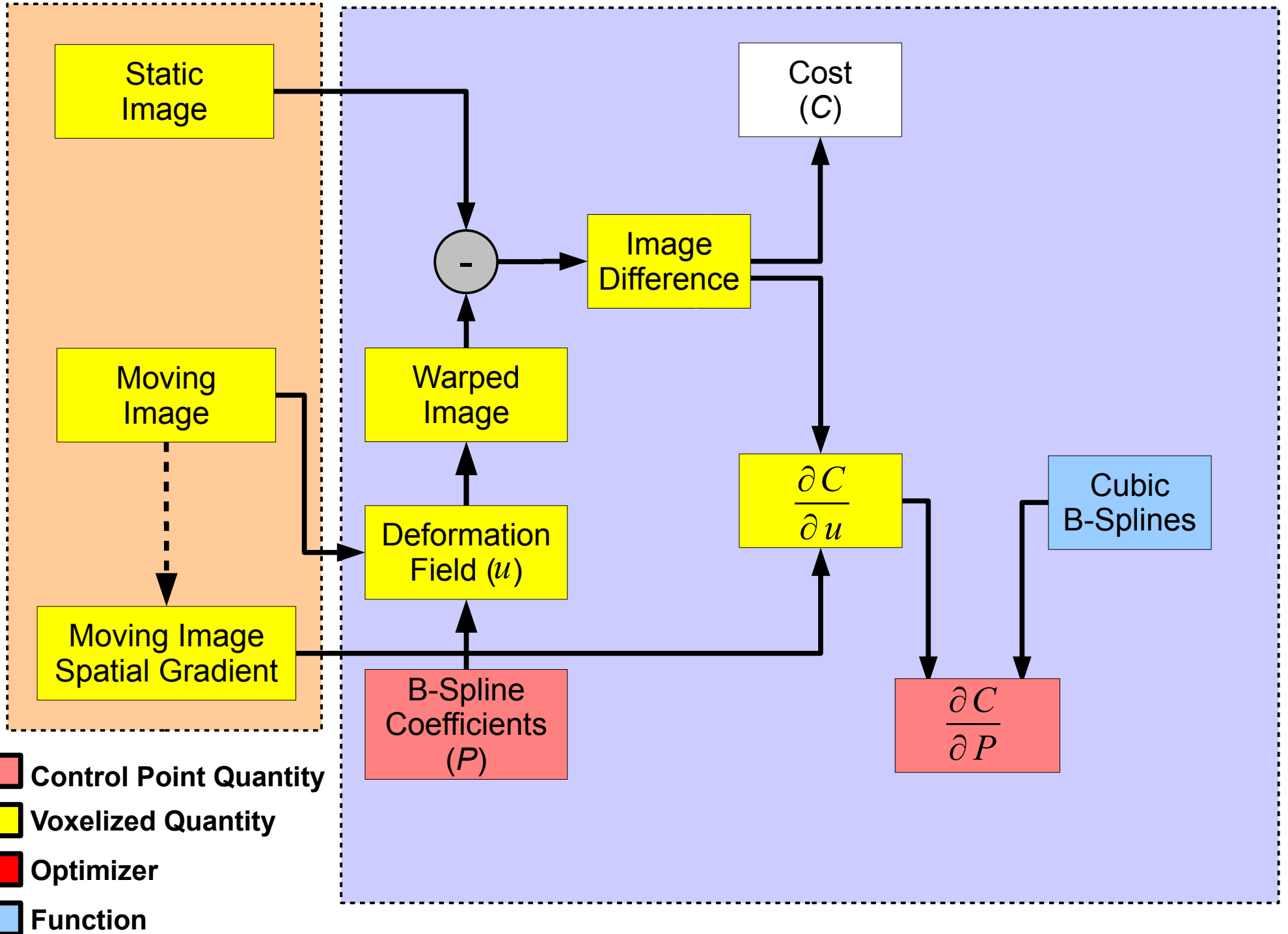
INPUTS

ITERATIVE REGISTRATION PROCESS



INPUTS

ITERATIVE REGISTRATION PROCESS



INPUTS

ITERATIVE REGISTRATION PROCESS

