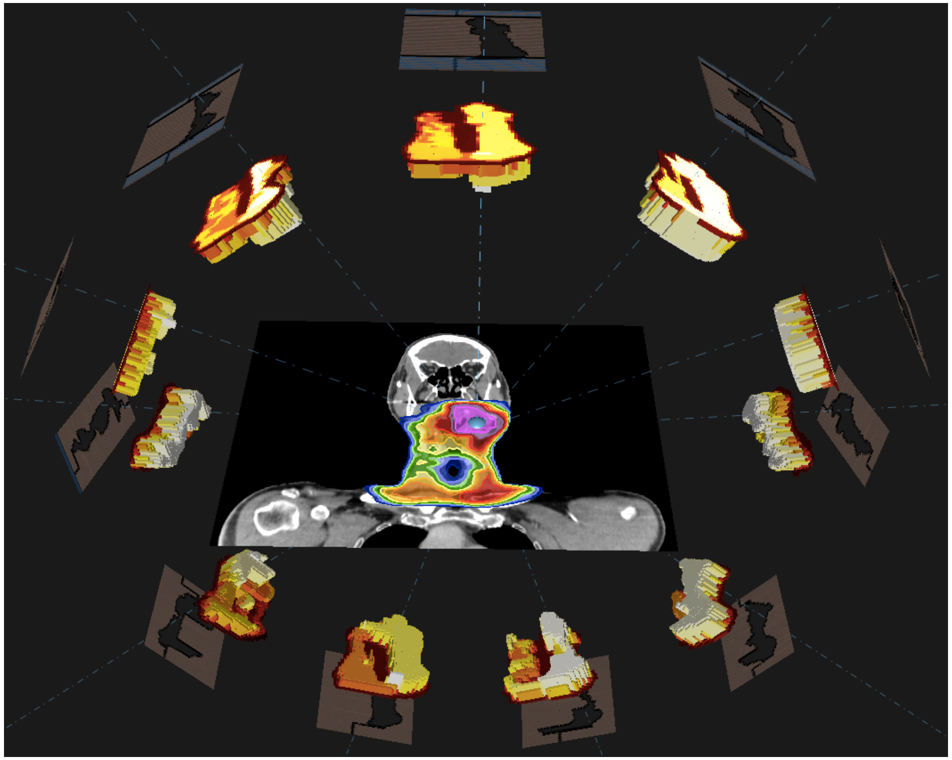
Intensity-modulated radiotherapy

Beyond fluence map optimization



Overview

So far:

Fluence map optimization

- Intensity-modulated proton therapy (IMRT)

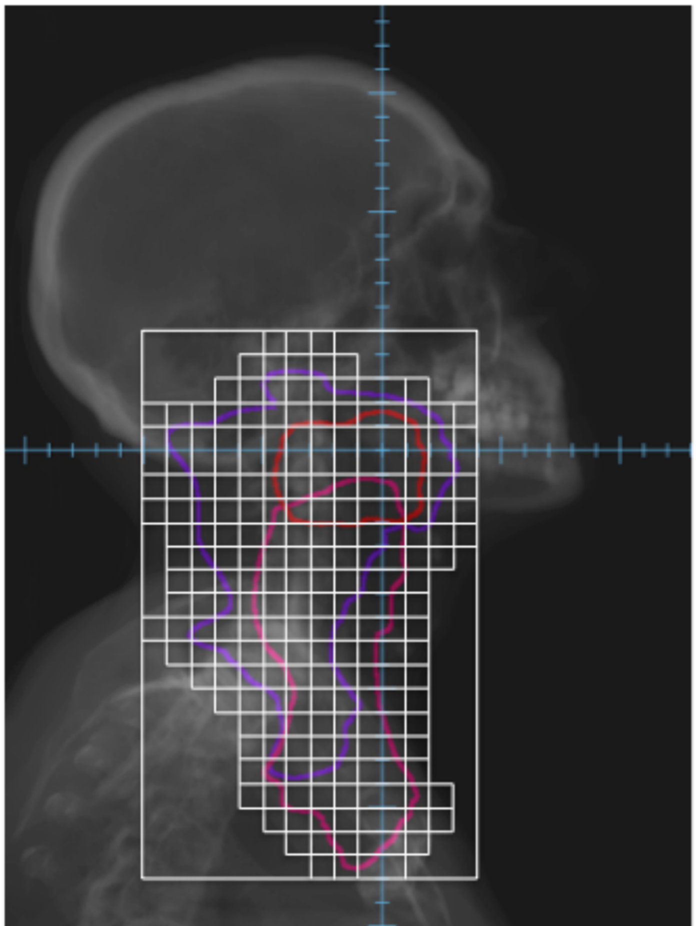
Now:

Extensions in IMRT planning

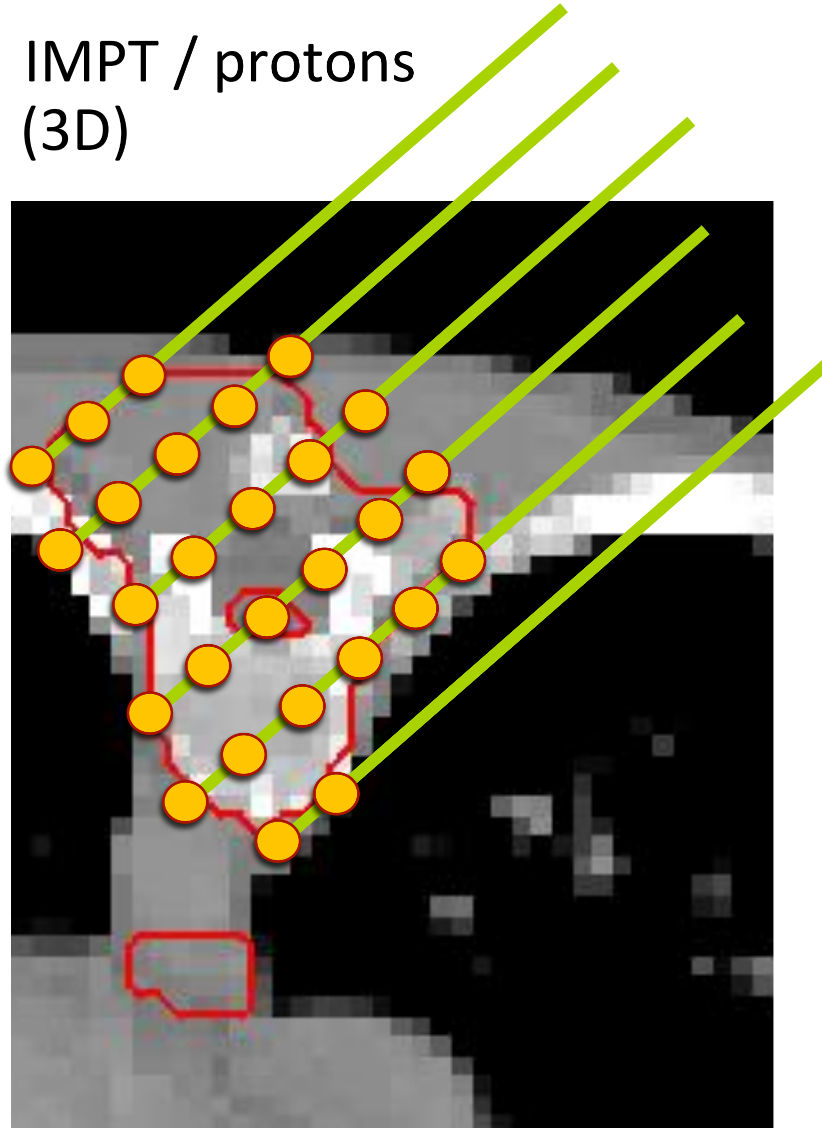
- leaf sequencing
- direct aperture optimization
- VMAT optimization

Fluence map

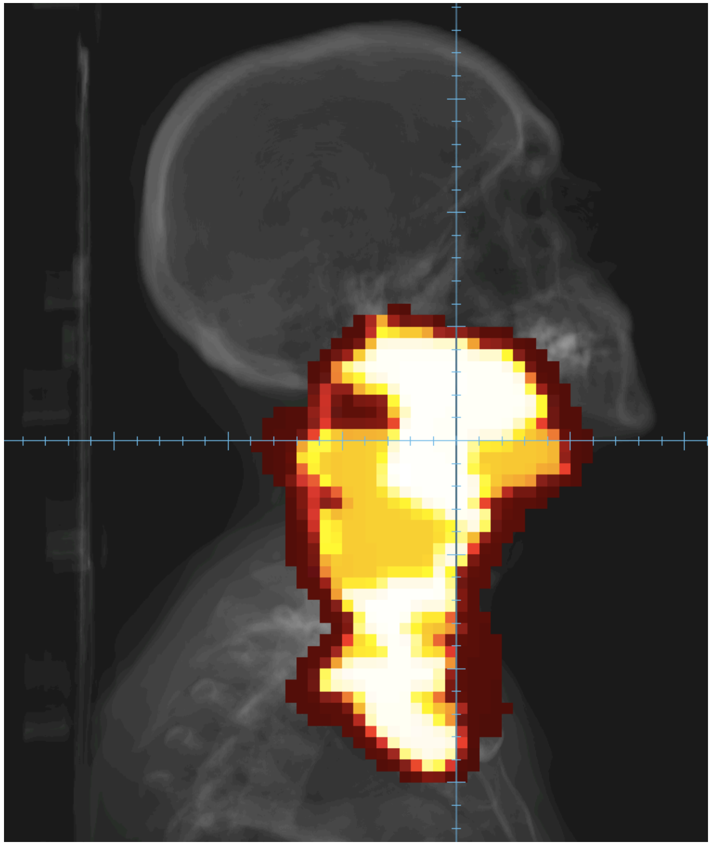
IMRT / photons
(2D)



IMPT / protons
(3D)

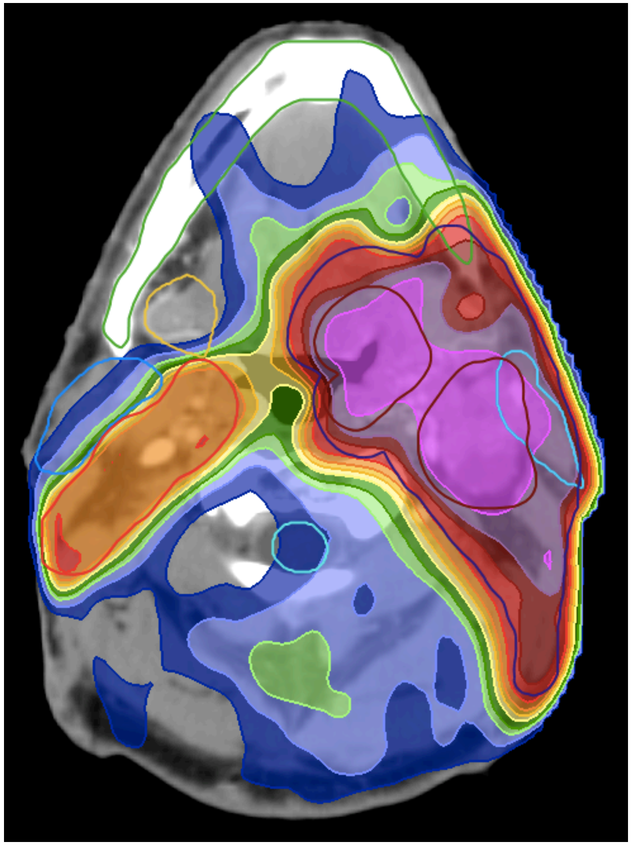


Overview



Fluence map

$$d_i = \sum_j x_j D_{ij}$$



Dose distribution

Overview

Traditional approach to IMRT planning:

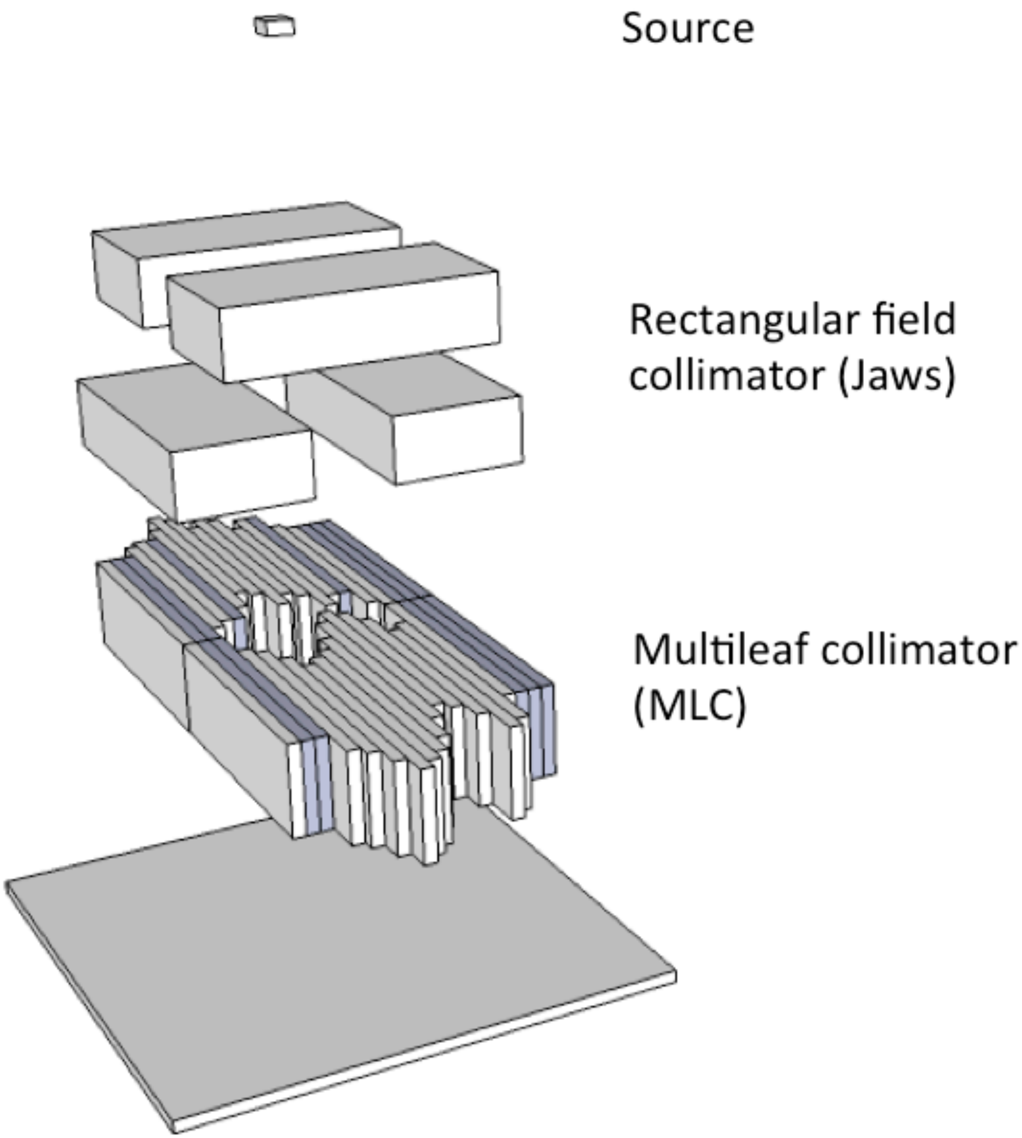
Two steps

1. Fluence map optimization
2. Leaf sequencing

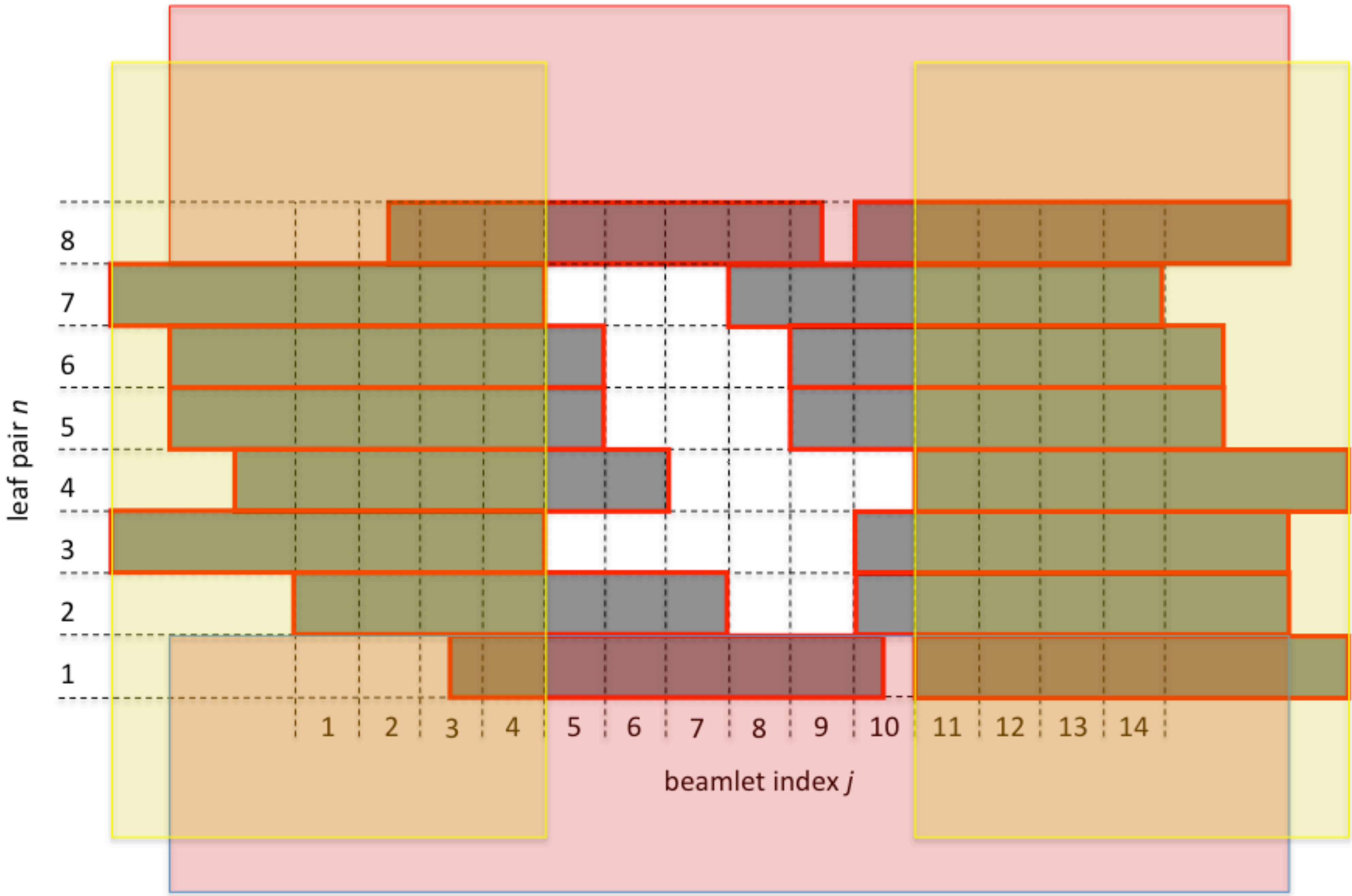
Third component

Direct Aperture Optimization (DAO)

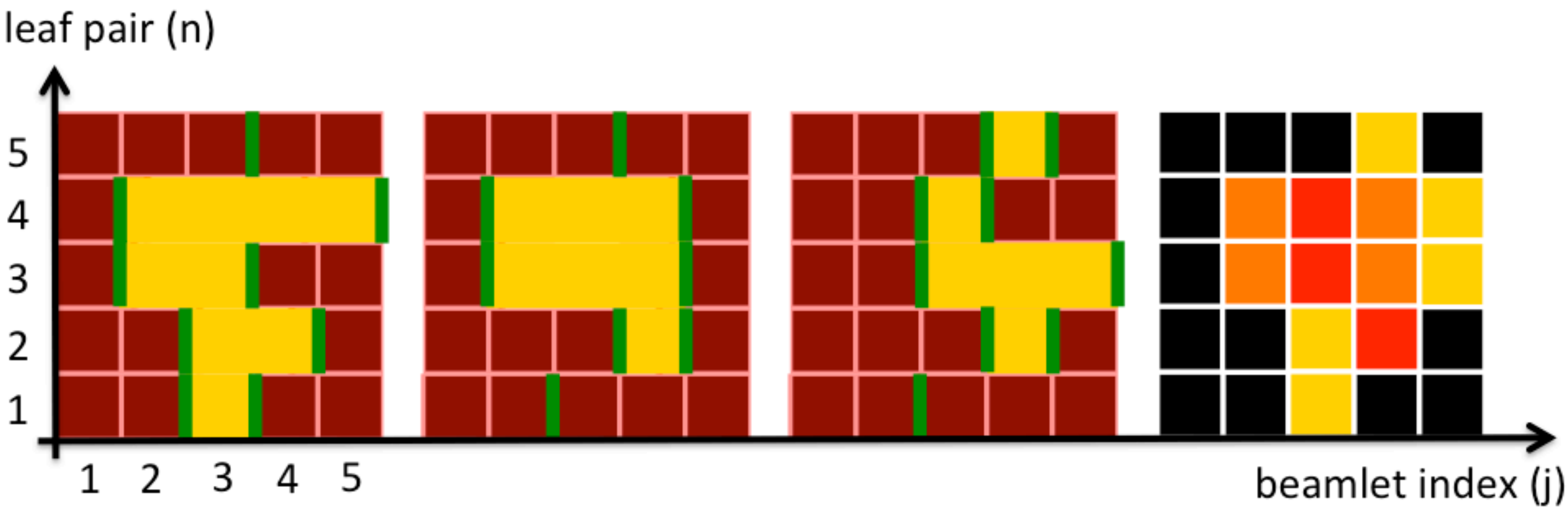
Beam collimation using MLC



Beam collimation using MLC



Superposition of multiple apertures yields an IMRT field



Leaf sequencing is the inverse problem

Find a set of MLC apertures that deliver an optimized fluence map

Leaf sequencing

- Leaf sequencing does not have a unique solution
  (trivial solution: deliver each beamlet individually)

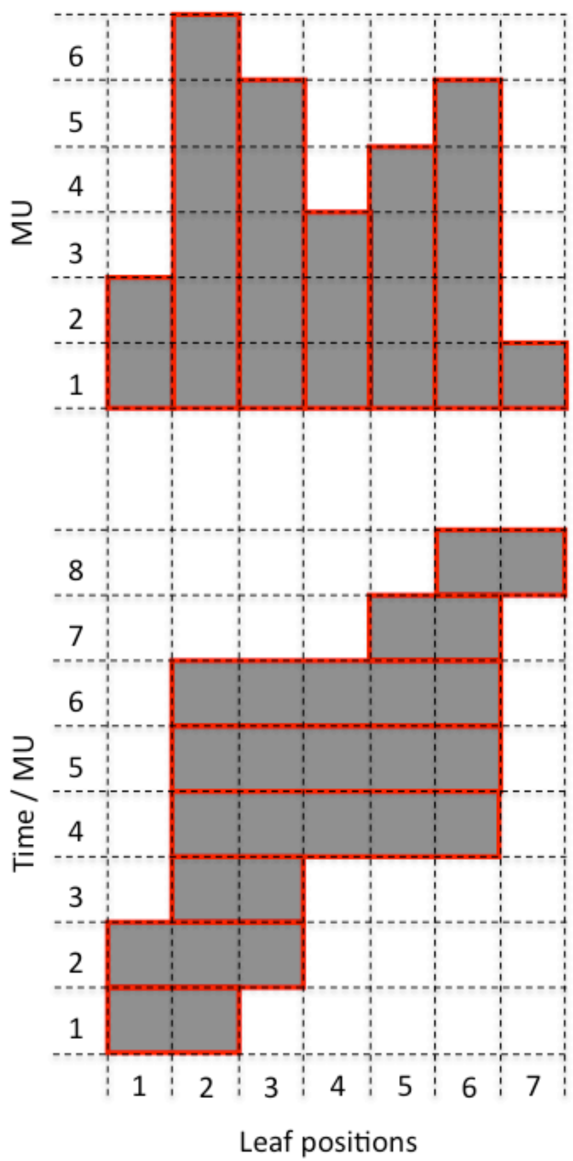
Goal: Find an efficient solution

- reproduce fluence map faithfully
- minimize number of apertures
- minimize total number of monitor units

Constructive method: Sliding window (minimizes total MU)

Discrete optimization methods: aim to minimize number of apertures

Sliding window sequencing



- consider discretized fluence map
- consider one MLC leaf pair
- leaves move uni-directionally
- left leaf positions:
  determined by positive gradients
- right leaf positions:
  determined by negative gradients
- total MU = sum of positive gradients

Direct aperture optimization

Limitations of Two-step approach:

- Poor dose calculation accuracy for fluence map optimization
  (use of pencil beam algorithm for Dose-influence matrix)

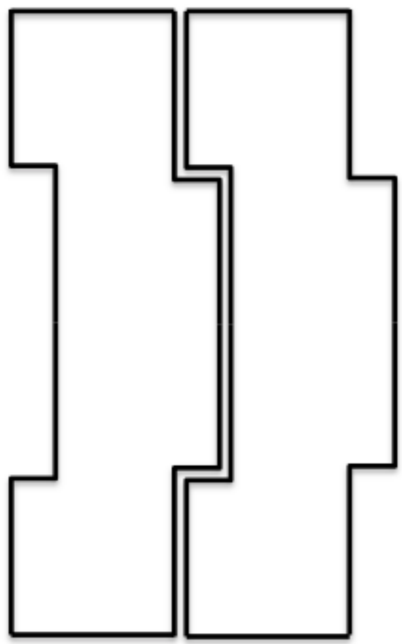
- Discrepancy between optimized fluence map and sequenced map
  (treatment plan with few apertures)

- discrete leaf positions limited to beamlet boundaries
  (benefit of fine tuning leaf positions at the target edge)

- Inherent limitations of the dose-influence matrix concept
  (example: tongue & groove effect)

Direct aperture optimization

Tongue & Groove design of MLC leaves:



- dose-influence matrix assumes linearity

  i.e. dose of beamlets delivered individually equals dose delivered by combined aperture

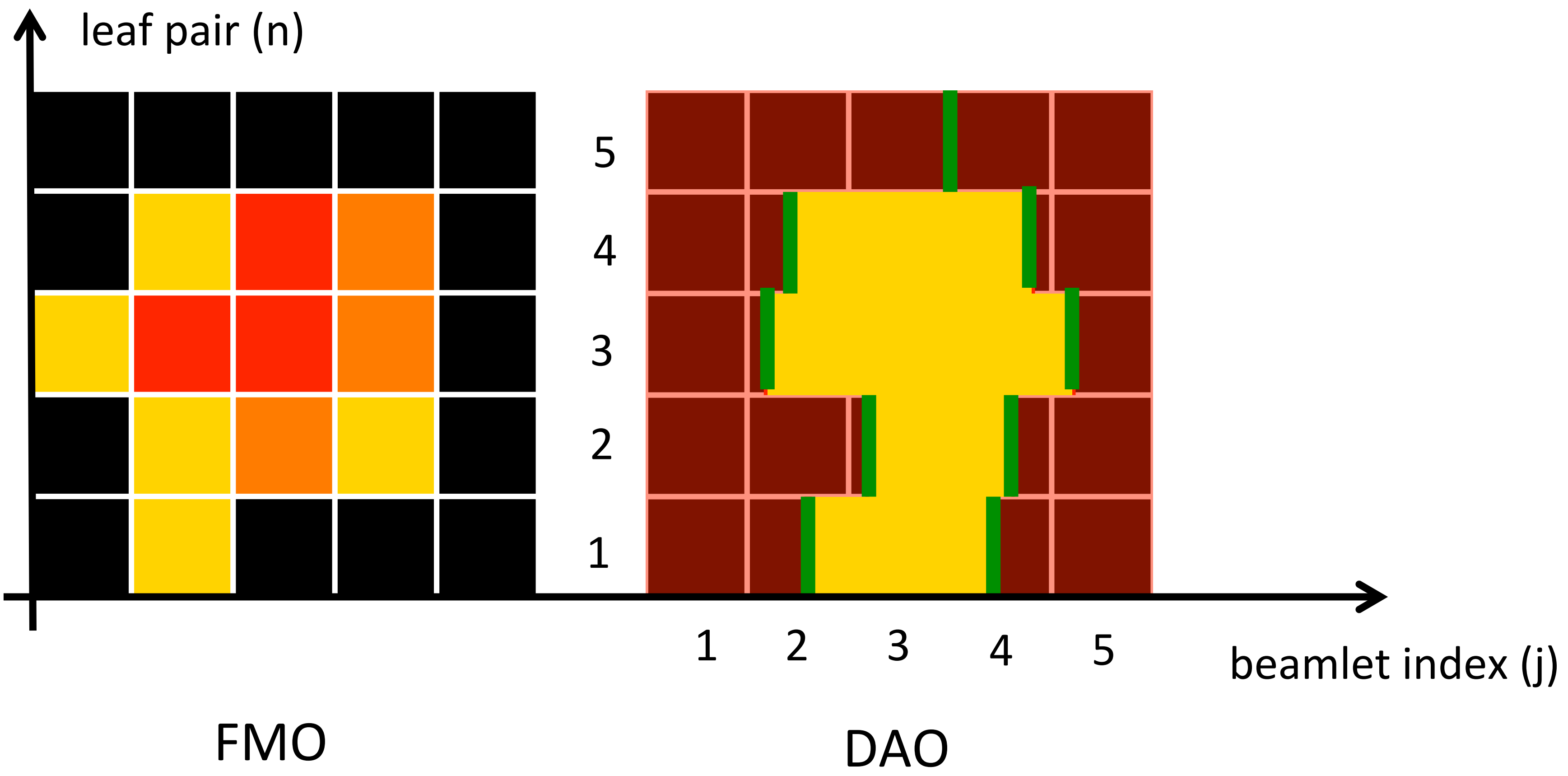
- not strictly true

center is blocked as soon as one leaf is closed

➔ center is underdosed if beamlets are delivered separately

Direct aperture Optimization (DAO)

= directly optimize shape and intensity of apertures



Direct aperture optimization

Fluence map optimization:

Dose is linear function of beamlet intensities

➔ efficient algorithms can find the global optimum

Fluence map optimization

<u>Objectives and constraints are 'nice' functions of the variables</u>

$$\underset{x}{\text{minimize}} \quad \underbrace{w_T \sum_{i \in T} \left(\sum_j x_j D_{ij} - 70 \right)^2 + w_H \sum_{i \in H} \sum_j x_j D_{ij}}$$

quadratic function of x_j

$$\text{subject to} \quad x_j \geq 0 \quad \forall j$$

$$\underbrace{\sum_j x_j D_{ij} \leq 40 \quad \forall i \in S}$$

linear functions of x_j

Direct aperture optimization

Fluence map optimization:

Dose is linear function of beamlet intensities

➔ efficient algorithms can find the global optimum

Direct aperture optimization:

Dependence of dose on leaf position is a smoothed step function

➔ highly non-convex optimization problem with local minima

How many apertures should each beam direction get?

➔ combinatorial aspect

Direct aperture optimization

Three common approaches

- stochastic search methods (simulated annealing)
  (commercialized by Prowess)

- gradient-based leaf position optimization
  (RayStation, Pinnacle)

- aperture generation methods

Gradient based DAO

How does dose depend on leaf positions?

$$d_i = \sum_k \sum_n y_k \Psi^i_{kn} \left(L_{kn}, R_{kn} \right)$$

dose in voxel i

sum over apertures k

sum over MLC rows n

dose contribution of MLC row n in aperture k to voxel i

leaf/right leaf positions

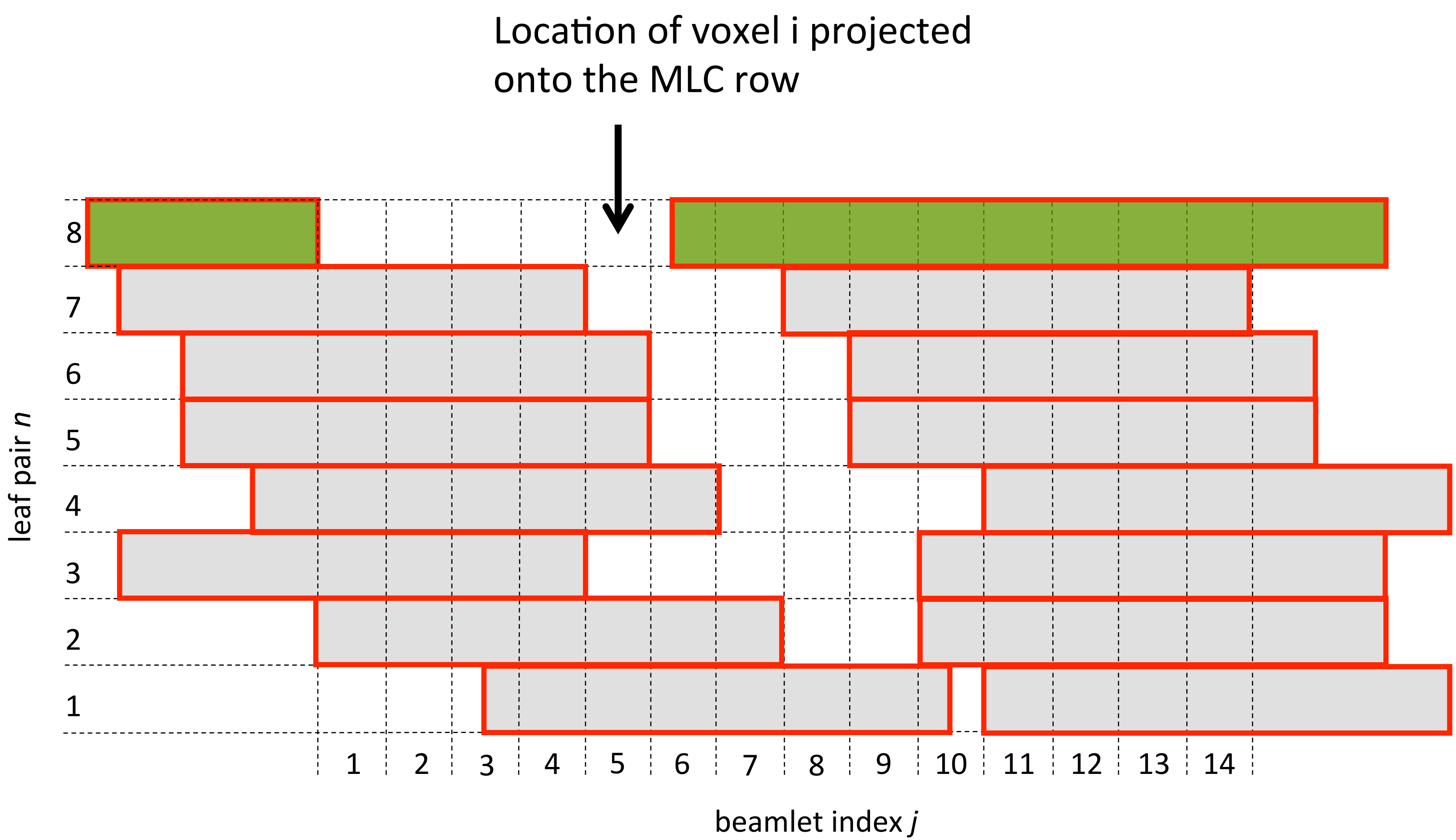
Dose dependence

- **Assume that left leaf is at the left-most position at the edge of the fluence map**

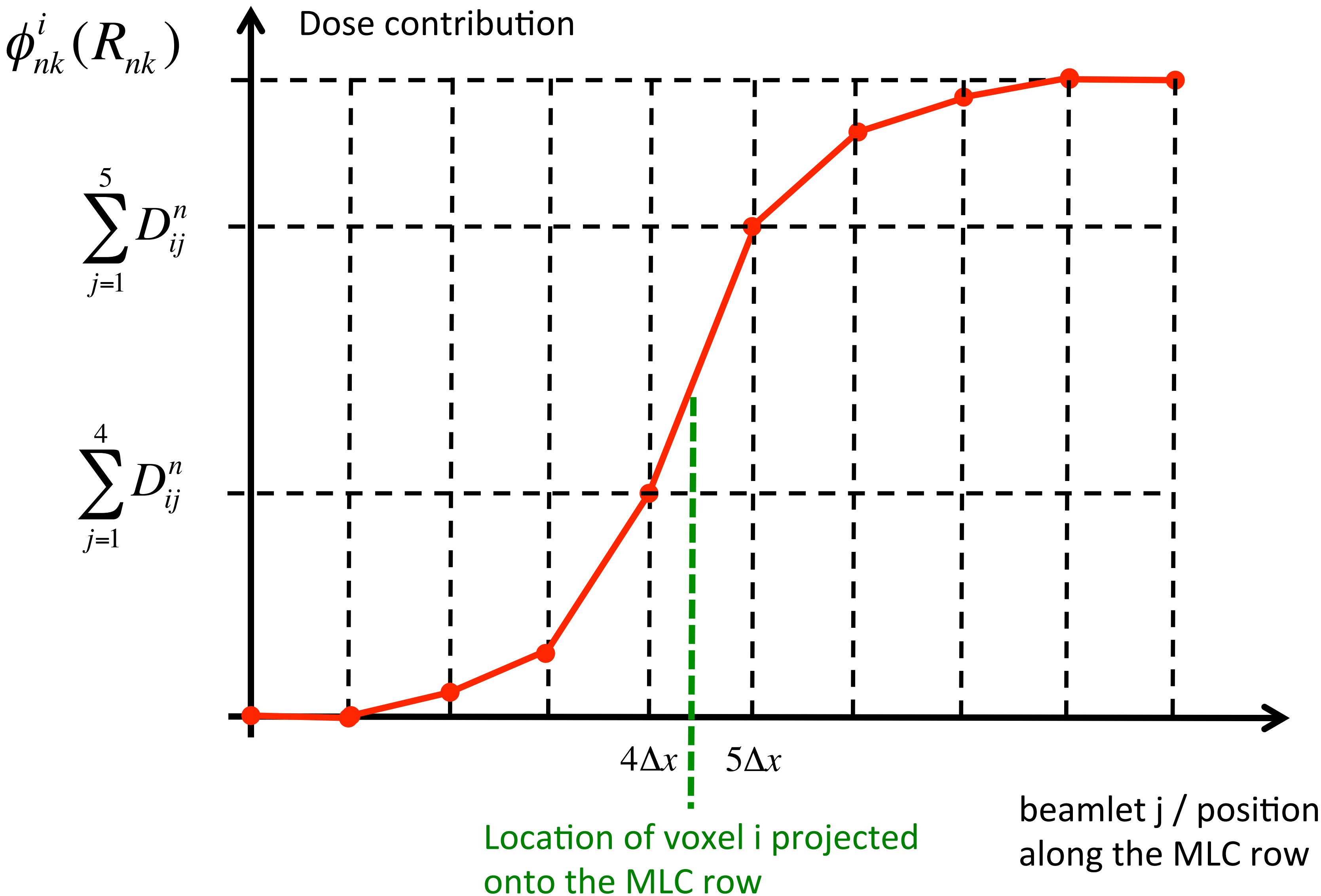
Determine dose contribution of MLC row as a function of the right leaf position:

$$\phi_{nk}^{i}(R_{nk})$$

Dose dependence



Dose dependence



Dose dependence

Dose contribution of MLC row n:

$\phi^i_{nk}(R_{nk})$ = piecewise linear function

(corners given by dose-influence matrix)

$$\Psi^i_{kn}\left(L_{kn}, R_{kn}\right) = \phi^i_{kn}\left(R_{kn}\right) - \phi^i_{kn}\left(L_{kn}\right)$$

contributions from beamlets that are not blocked by the right leaf, assuming the left leaf is at the edge of the field

subtract dose contributions from beamlets blocked by the left leaf

Direct aperture optimization

Treatment plan optimization

$$\underset{x}{\text{minimize}} \quad w_T \sum_{i \in T} \left(d_i - 70\right)^2 + w_H \sum_{i \in H} d_i$$

minimize deviation from 70 Gray in the tumor

minimize dose in healthy tissues

$$\text{subject to} \quad x_j \geq 0 \quad \forall j$$

fluence cannot be negative

Direct aperture optimization

Treatment plan optimization

$$\underset{x}{\text{minimize}} \quad w_T \sum_{i \in T} (d_i - 70)^2 + w_H \sum_{i \in H} d_i$$

minimize deviation from 70 Gray in the tumor

minimize dose in healthy tissues

~~subject to $x_j \geq 0 \quad \forall j$~~ ~~fluence cannot be negative~~

Direct aperture optimization

Treatment plan optimization

$$\underset{y,L,R}{\text{minimize}} \quad w_T \sum_{i \in T} \left(d_i - 70\right)^2 + w_H \sum_{i \in H} d_i$$

$$d_i = \sum_k \sum_n y_k \Psi_{kn}^i \left(L_{kn}, R_{kn}\right)$$ (dose in voxel i)

$$L_{kn} \le R_{kn} \quad \forall n,k$$ (leaves cannot cross)

$$y_k \ge 0 \quad \forall k$$ (positive aperture weights)

Gradient based optimization

Derivative with respect to aperture weight:

$$d_i = \sum_k \sum_n y_k \Psi^i_{kn}\left(L_{kn}, R_{kn}\right)$$

$$\frac{\partial f}{\partial y_k} = \sum_i \frac{\partial f}{\partial d_i}\frac{\partial d_i}{\partial y_k} = \sum_i \left[\frac{\partial f}{\partial d_i}\sum_n \Psi^i_{kn}\left(L_{kn}, R_{kn}\right)\right]$$

Gradient based optimization

Derivative with respect to leaf positions:

$$d_i = \sum_k \sum_n y_k \Psi_{kn}^i \left(L_{kn}, R_{kn} \right)$$

$$\Psi_{kn}^i \left(L_{kn}, R_{kn} \right) = \phi_{kn}^i \left(R_{kn} \right) - \phi_{kn}^i \left(L_{kn} \right)$$

$$\frac{\partial f}{\partial R_{kn}} = \sum_i \frac{\partial f}{\partial d_i} \frac{\partial d_i}{\partial R_{kn}} = \sum_i \left[\frac{\partial f}{\partial d_i} y_k \frac{\partial \phi_{kn}^i \left(R_{kn} \right)}{\partial R_{kn}} \right]$$

$$\propto D_{ij}^n$$

j = beamlet index where leaf edge is positioned

Volumetric modulated arc therapy (VMAT)

- Continuous delivery mode:

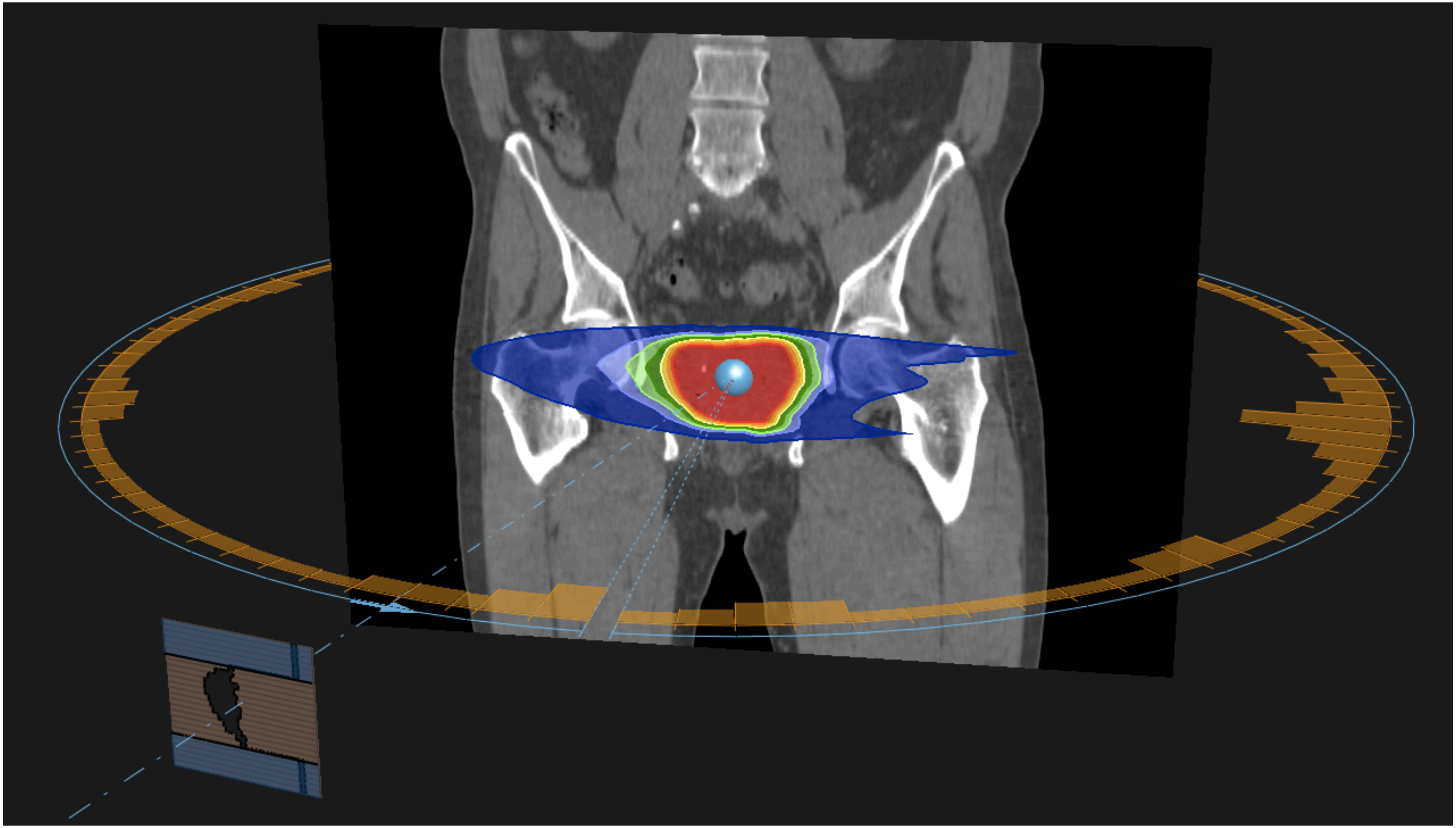
  Radiation beam is constantly on while gantry and MLC leaves move

- Variables (conceptually):
  - leaf positions as a function of time
  - gantry angle as a function of time
  - dose rate as a function of time

- Variables (in practice): (driven by DICOM specification)

  Determine one aperture every 2 degrees

VMAT optimization



VMAT optimization

VMAT optimization methods

Ref: Unkelbach et al, Med Phys, 2015,
'Optimization approaches to volumetric modulated arc therapy planning'

VMAT approaches reuse the concepts from IMRT planning

- fluence map optimization
- (arc) sequencing
- direct aperture optimization

- **Approaches differ in the exact implementation of each step, and on the step they rely on most**

VMAT optimization

Example: Prostate case treated in a single 360 degree arc

Goal:
- divide arc into 180 sectors of 2 degree lengths
- assign one aperture (control point) to each sector
- neighboring apertures should be similar

Raystation, Pinnacle, Monaco:

Three-step approach (largely rely on DAO step)

1. Fluence map optimization
2. Arc sequencing
3. Direct aperture optimization

VMAT optimization

Step 1: Perform fluence map optimization at 20 equispaced angles

Step 2: approximate each fluence map through 9 apertures

(yields 180 control points, one every 2 degrees)

typical approach: sliding window sequencing

Why?

VMAT optimization

Step 1: Perform fluence map optimization at 20 equispaced angles

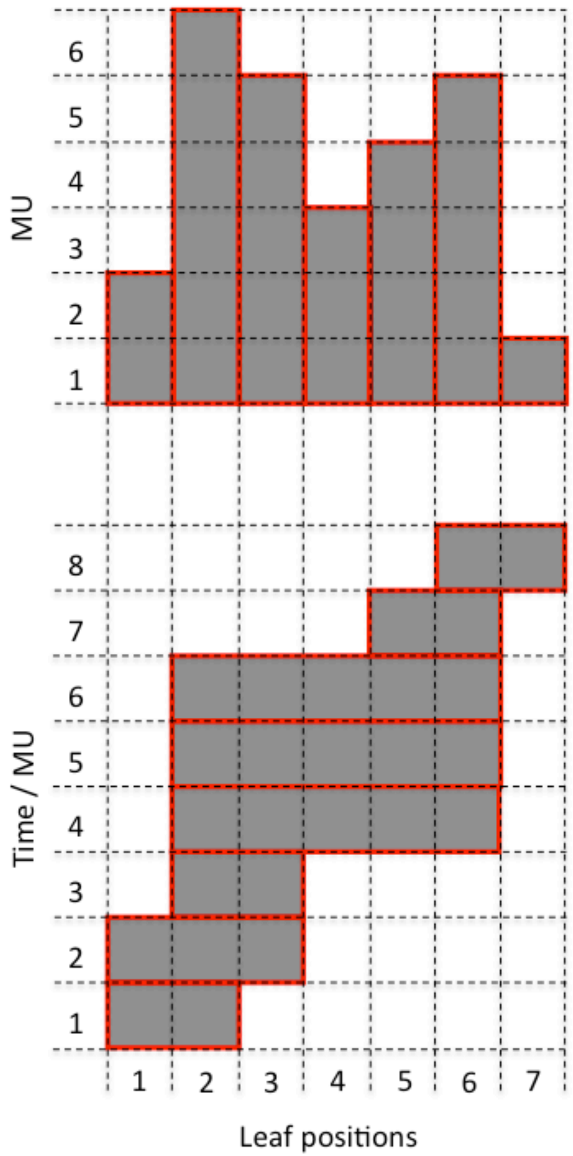
Step 2: approximate each fluence map through 9 apertures

(yields 180 control points, one every 2 degrees)

typical approach: sliding window sequencing

Why? Naturally yields ordered apertures. Unidirectional leaf motion makes subsequent apertures similar.

Sliding window sequencing



VMAT optimization

Step 1: Perform fluence map optimization at 20 equispaced angles

Step 2: approximate each fluence map through 9 apertures

(yields 180 control points, one every 2 degrees)

typical approach: sliding window sequencing

Why? Naturally yields ordered apertures. Unidirectional leaf motion makes subsequent apertures similar.

Step 3: Perform gradient based DAO

one aperture at each of 180 gantry angles

VMAT optimization

DAO step yields a DICOM VMAT plan, specified by

for each control point:

- all leaf positions
- gantry angle
- (couch and collimator angle)
- total MU delivered up to this control point

this does not contain time!

Linac machine controller translates DICOM into trajectories

(TPS relies on assumptions to estimate treatment time)

How do aperture weights relate to gantry speed and dose rate?

$$y_k = \frac{\Delta_k \delta_k}{s_k}$$

aperture weight [MU]

angular distance between control points [degree]

gantry speed [degree/s]

dose rate [MU/s]

large aperture weight realized by high dose rate or low gantry speed

VMAT optimization

Constrain maximum leaf travel for efficiency:

assuming we want to deliver a 360 degree arc in one minute (maximum gantry speed)

control point spacing: $\Delta_k = 2°$
gantry speed: s = 6° / second

➔ ⅓ second per control point

maximum leaf speed: v = 6 cm / second

➔ maximum leaf travel between control points is $\Delta = 2$ cm

Direct aperture optimization

$$\underset{y,L,R}{\text{minimize}} \quad w_T \sum_{i\in T} \left(d_i - 70\right)^2 + w_H \sum_{i\in H} d_i$$

$$d_i = \sum_k \sum_n y_k \Psi^i_{kn}\left(L_{kn}, R_{kn}\right)$$ (dose in voxel i)

$$L_{kn} \le R_{kn} \quad \forall n,k$$ (leaves cannot cross)

$$y_k \ge 0 \quad \forall k$$ (positive aperture weights)

$$\left|R_{kn} - R_{(k+1)n}\right| \le \Delta$$ (constrain maximum leaf travel between apertures)